$$\therefore\left(\frac{s+t}{C}+\frac{C+sn}{Cmn}\right)y=\left(\frac{C+s+t}{C}+\frac{s}{Cm}\right)x\ldots(3);\ y=\frac{(C+sn)x}{sn}\ldots(4).$$

(4) in (3) gives $C=mn(sn-s-t)-2sn$.

292. Proposed by REV. R. D. CARMICHAEL, Anniston, Ala.

Find the sum of the series $1^2+5^2+14^2+30^2+\ldots+[\frac{1}{6}n(n+1)(2n+1)]^2$.

Solution by THEODORE L. DeLAND, Treasury Department, Washington, D. C.

The differences and the terms of this special series may be arranged as follows for the first seven terms:

$u_1=1^2$		5^2		14^2		30^2		55^2		91^2		140^2
$u_1=1$		25		196		900		3025		8281		19600
$\Delta^1u_1=$	24		171		704		2125		5256		11319	
$\Delta^2u_1=$		147		533		1421		3131		6063		
$\Delta^3u_1=$			386		888		1710		2932			
$\Delta^4u_1=$				502		822		1222				
$\Delta^5u_1=$					320		400					
$\Delta^6u_1=$						80						

Compute the series for ten terms, or more, and it will be found that Δ^6u_1 are all 80, or constant, therefore all the higher differences vanish. To sum the series we have the value of the leading term and the six leading differences. I have given a general formula for S_n, on page 163, of THE AMERICAN MATHEMATICAL MONTHLY for August-September, 1906, see equation (E). We have:

$$S_n=nu_1+\frac{n(n-1)}{2}\Delta^1u_1+\frac{n(n-1)(n-2)}{3!}\Delta^2u_1+\ldots$$

$$+\frac{n(n-1)\ldots(n-6)}{7!}n^6u_1\ldots(1).$$

From the problem and the above table we have: $u_1=1$, $\Delta'=24$, $\Delta^2=147$, $\Delta^3=386$, $\Delta^4=502$, $\Delta^5=320$, and $\Delta^6=80$. Substitute numerical values in (1), expand the terms, consolidate like terms, reduce, and we have:

$$S_n=\frac{20n^7+140n^6+371n^5+455n^4+245n^3+35n^2-6n}{1260}\ldots(2),$$

$$=\tfrac{1}{1260}[n(n+1)(n+2)(2n+1)(2n+3)(5n^2+10n-1)].$$

Also solved by E. B. Escott, and G. B. M. Zerr. Professor Escott solved the problem by putting the general term equal to $A+Bn+Cn(n+1)+...+Gn(n+1)(n+2)(n+3)(n+4)(n+5)$. Then by letting $n=0$, -1, -2, etc., he determines A, B, ..., G. The general term is thus reduced to five terms of the form $n(n+1)...(n+r-1)$. Since the sum of a series whose general term is $n(n+1)(n+2)...(n+r-1)$ is $[n(n+1)...(n+r-1)]/[r+1]$ finds the sum which agrees with that obtained by Mr. DeLand.

Dr. Zerr decomposed the general term in a similar way and after summing the five similar series thus arising he gets the same result as that given above.

GEOMETRY.

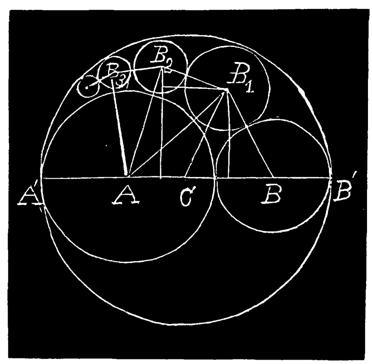
326. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The circle C of radius pR encloses the circles A_1, B_1 of radii R and $(p-1)R$, respectively; the circle B_1 is tangent to A_1, B_1, C_1; the circle B_2 is tangent to A, B_1, C; the circle B_3 to A, B_2, C, ..., B_n to A, B_{n-1}, C. Find the radius of the circle B_n.

Solution by the PROPOSER.

First find the locus of centers of circles tangent to A and C, taking A' the point of contact of A and C as the origin.

Let r, r_1, r_2, ..., r_n be the radii of B, B_1, ..., B_n, respectively; r'=radius of any circle tangent to circles whose centers are A, C; and x, y co-ordinates of its centers. Then $(r'+R)^2-(R-x')^2=(pR-r')^2-(pR-x')^2=y'^2$...(1).



$$\therefore r=\frac{(p-1)x'}{p+1}...(2), \text{ and } x'=\frac{(p+1)r'}{p-1}...(3).$$

Substituting the value of x' in (1), we have

$$(R+r')^2-(R-\frac{p+1}{p-1}r')^2=y'^2.$$

$$\therefore y'=\frac{2}{p-1}\sqrt{[p(p-1)Rr'-pr'^2]}.$$ Since $r=(p-1)R$ and $x=R(p+1)$, $(p-1)R$ is of the form $\dfrac{p(p-1)R}{0^2(p-1)^2+p}$.

2. Find r_1. Join centers of A, B, and C with B_1, B_2, ..., B_n. Draw perpendiculars from centers B_1, B_2, ..., to the diameter of C passing through A'.

$$(r+r'_1)^2-(x-x_1)^2=(pR-r_1)^2-(pR-x_1)^2...(4). \quad x_1=\left(\frac{p+1}{p-1}\right)r_1.$$

Substitute the values of x, r, x_1 in (4); whence

$$4Rr_1(p^2-p+1)=(p-1)pR^2. \quad \therefore r_1=\frac{p(p-1)R}{(p-1)^2+p}=\frac{p(p-1)R}{1^2(p-1)^2+p}.$$